

COMPUTATIONAL NUMBER THEORY-I

PREPARATORY INSTRUCTIONS

1. PRE-REQUISITES FOR THE LECTURE

As the name suggests, the course will involve using computers. A lot. My personal preference is to use one of the following (in descending order of priority):

- (1) Pari/GP
- (2) SageMath
- (3) Mathematica, C++, Haskell, Maple, Julia

Pari/GP is software written for the sole purpose of arithmetic geometry and number theory. SageMath parses well will Pari/GP and also has standalone support. Mathematica, C++, Haskell, Maple, Julia all have varying levels of support with number theory functions and libraries. At the end of the day, each program has its advantages and disadvantages but if you can get your code running in any of the above languages, you should be all good to go. That being said, Pari/GP is what I will be using. But you're welcome to use any of the above. (My experience with number theory on Julia and Maple is small but growing so I will be happy to learn this with you should you choose to go down that path).

1.1. Disclaimers - Read Carefully:

- (1) Installation of software and updates above is usually standard and harmless to your computer.
- (2) However, installing any and all software pertaining to the course is at your own risk. Neither I nor the MPI-MiS are responsible for any damage incurred due to faulty installations of software, any steps undertaken in that regard, or due to software updates.
- (3) The following instructions are usually standard. However, you are **strongly encouraged** to verify if whether the commands are indeed appropriate for your system. For example: `pip` versus `pip3`.

1.2. **Using Pari/GP.** Pari/GP is a remarkably powerful number theory and arithmetic geometry oriented software which seems a bit daunting at first but there are several reasons to use it:

- (1) It has almost all the routines and algorithms in number theory and arithmetic geometry
- (2) It has very effective and smart cache management; you can do a lot with very little memory!
- (3) It is fast and parallelizable
- (4) Continual, systematic and focussed developments from the Gods of computational number theory
- (5) Very flexible to including new, user developed packages
- (6) Uniformity of all libraries in syntax and command names (biggest issue with number theory on SageMath).

There are many ways to install Pari/GP.

Installation images: <https://pari.math.u-bordeaux.fr/download.html>

Other relevant packages: <https://pari.math.u-bordeaux.fr/packages.html>. If you install via these packages, make sure that you are able to call Pari/GP from the terminal. Alternatively you can try one of the following:

```

1 Debian: $ sudo apt install pari-gp
2 Mac: $ brew install pari
3

```

Note that Pari/GP announced an update recently and this is not yet on the Debian list of approved packages. Once you have this, you can call Pari/GP on the terminal via

```

1 $ gp
2
3 GP/PARI CALCULATOR Version 2.17.0 (released)
4 arm64 running darwin (aarch64/GMP-6.3.0 kernel) 64-bit version
5 compiled: Sep 28 2024, Apple clang version 15.0.0 (clang-1500.3.9.4)
6 threading engine: pthread, nbthreads = 12
7 (readline v8.2 enabled, extended help enabled)
8
9 Copyright (C) 2000-2024 The PARI Group
10
11 PARI/GP is free software, covered by the GNU General Public License, and
12 comes WITHOUT ANY WARRANTY WHATSOEVER.
13 Type ? for help, \q to quit.
14 Type ?18 for how to get moral (and possibly technical) support.
15
16 parisize = 8000000, primelimit = 1048576, factorlimit = 1048576
17 ?
18

```

Type `\q` or `quit` to quit Pari/GP.

Pro's of running Pari this way:

- (1) Command completion
- (2) Scripting
- (3) Faster

Con's of running Pari this way:

- (1) You *need* to save your input commands and workflow via copy paste or some other fancy technique
- (2) You're restricted to the same typing restrictions as on a terminal

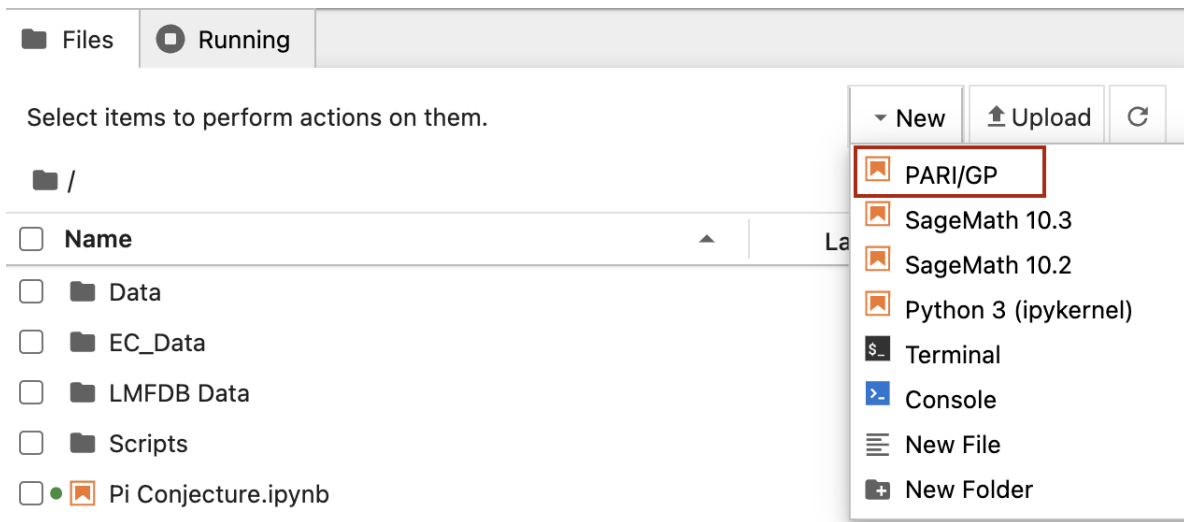
You can also run Pari/GP on a jupyter notebook in SageMath. You should have SageMath installed already. If you don't: This can be done by following steps on <https://doc.sagemath.org/html/en/installation/index.html>. To install Pari/GP on a jupyter environment:

```

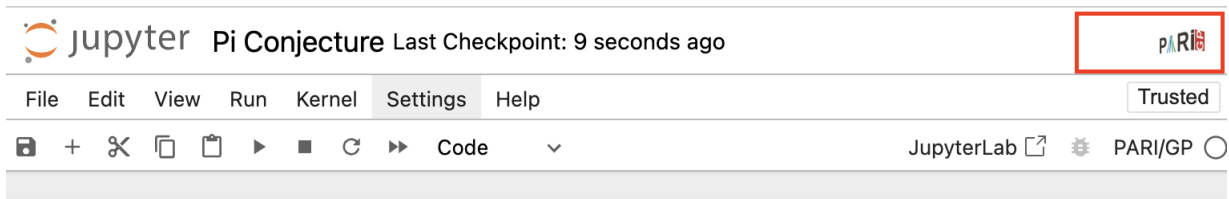
1 $ sage
2 $ pip install pari-jupyter
3 # Restart the sage kernel
4 $ exit()
5
6 $ cd /to/your/directory
7 $ sage -n jupyter

```

Now you should see the possibility to initialize a new Pari/GP notebook:



at which point you will see



1.3. **Using SageMath.** You are of course welcome to use SageMath too. In fact it has in-built Pari/GP parsing too, and this can be found in the tutorial here: <https://doc.sagemath.org/html/en/tutorial/interfaces.html#gp-pari>

Pro's of running Pari this way:

- (1) Commands are saved
- (2) You get the best of Pari/GP and SageMath

Con's of running Pari this way:

- (1) No command completion
- (2) Manuals are limited